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Deterministic learning from control of nonlinear systems with disturbances

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Abstract

In this paper, we investigate deterministic learning in environments with disturbances. We will show that for a class of uncertain nonlinear systems with bounded disturbances, by using an appropriately designed adaptive neural controller, the disturbances are attenuated and the system output tracks a periodic orbit in finite time. As radial basis function (RBF) neural networks (NN) are employed, this leads to the satisfaction of a partial persistence of the excitation (PE) condition. By using the uniform complete observability (UCO) technique, it is analyzed that partial estimated NN weights will converge to a neighborhood of zero, with the size of the neighborhood depending on the amplitude of disturbances as well as on the control gains. Locally-accurate approximation of unknown system dynamics can still be achieved in the stable NN control process. The approximation error level is influenced by the amplitude of disturbances. The obtained knowledge of system dynamics can be reused in another control process towards stability and improved performance. Simulation studies are included to demonstrate the effectiveness of the approach.

Keywords: Deterministic learning; Adaptive NN control; Disturbance; Partial PE condition; Uniform complete observability

1. Introduction

Neural network (NN)-based identification and control for uncertain systems have attracted tremendous interest in the past two decades [1–14]. Initially motivated by the learning and control abilities of human beings, NN control should at least possess two properties: (i) be capable of learning good knowledge online through stable closed-loop control processes; and (ii) be capable of exploiting the learned knowledge in the same or similar control tasks towards stability and improved control performance [15,16,20]. These two properties, however, have been less investigated in the control literature. Specifically, the property of learning knowledge from stable control processes involves convergence of the estimated weights of neural networks to their true or optimal values, which is not easily achieved due to the difficulty in satisfying the persistence of the excitation (PE) condition.

Recently, a deterministic learning mechanism was presented [15], by which an appropriately designed adaptive neural controller is capable of learning the unknown system dynamics in a closed-loop control process. The deterministic learning is achieved according to the following elements: (i) tracking control of the system states to a periodic reference orbit; (ii) satisfaction of a partial PE condition by using localized RBF networks; (iii) exponential stability of the closed-loop system along the tracking orbit, including the convergence of certain neural weights to their optimal values; and (iv) accurate approximation of the unknown dynamics by the RBF network in a local region along the periodic tracking orbit. A neural learning control

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scheme was also proposed which can effectively utilize the learned knowledge towards improved control performance. With deterministic learning, not only is the learning property implemented through stable control processes, but the control performance is also improved with the utilization of the learned knowledge.

In this paper, we investigate deterministic learning from neural control of a class of uncertain nonlinear systems with bounded disturbances. It is known that when systems are subjected to disturbances, stability and tracking performance will deteriorate. Moreover, the existence of bounded disturbances will make the analysis of parameter convergence (i.e. learning) much more complicated. We will show that by using an appropriately designed adaptive neural controller, the disturbances are attenuated in the sense that the system output tracks a periodic orbit in finite time. With good tracking performance, the inputs of the RBF network can still satisfy a partial PE condition, and exponential stability of the nominal part of a closed-loop error system is obtained. By using the uniform complete observability (UCO) technique [17,22], it is analyzed that the estimated NN weights will converge to a neighborhood of zero, with the size of the neighborhood depending not only on the amplitude of disturbances but also on the control gains. Locally-accurate approximation of unknown system dynamics can still be achieved in the stable NN control process. The approximation error level is influenced by the amplitude of disturbances. The obtained knowledge of system dynamics is stored in constant RBF networks, and can be reused in another control process such that stability with good tracking performance can be guaranteed.

The rest of this paper is organized as follows. Preliminary results are given in Section 2. The main result of learning from the control with disturbances is presented in Section 3. A numerical simulation to illustrate the effectiveness of our approach is given in Section 4. The conclusion is given in Section 5.

2. Preliminaries

2.1. Localized RBF NN and PE condition

2.1.1. Universal approximation

It has been proven in Ref. [21] that any continuous function $f(Z)$, defined in any compact set $\Omega_Z \in \mathbb{R}^q$, can be approximated by the following RBF NNs with sufficiently large nodes and appropriately placed centers

$$f(Z) = W^{*\mathrm{T}} S(Z) + \varepsilon(Z), \quad \forall\, Z \in \Omega_Z \tag{1}$$

where W^* is the ideal weight vector, $S(Z)$ is the basis function vector and $\varepsilon(Z)$ is the approximation error assumed to satisfy $|\varepsilon(Z)| < \varepsilon^*$ with a small constant $\varepsilon^* > 0$.

2.1.2. Spatially localized approximation

For any point Z_ζ or any trajectory $\phi_\zeta(Z_\zeta(t))$ in compact set $\Omega_Z, f(Z_\zeta)$ can be approximated by using a limited number of neurons located at a neighborhood of the point, or a local region along the trajectory, i.e.,

$$f(Z_\zeta) = W_\zeta^{*\mathrm{T}} S_\zeta(Z_\zeta) + \varepsilon_\zeta(Z_\zeta) \tag{2}$$

where $S_\zeta(Z_\zeta) = [s_{\zeta 1}(Z_\zeta), \ldots, s_{\zeta m}(Z_\zeta)]^{\mathrm{T}} \in \mathbb{R}^m$ is a subvector of $S(Z_\zeta), s_{\zeta i}(Z_\zeta) > \tau,\ i = 1, \ldots, m$, with τ being a small positive constant, $\varepsilon_\zeta(Z_\zeta)$ is the approximation error with $\| \varepsilon_\zeta(Z_\zeta)| - |\varepsilon(Z_\zeta)\|$ being small.

2.1.3. PE of localized RBF NN

The definition of the PE condition is given as follows [18].

Definition 1 (*PE condition*)**.** A piecewise continuous, uniformly bounded, vector-valued function $S_\zeta : [0, \infty) \to \mathbb{R}^m$ is said to satisfy the PE condition, if there exist positive constants α_1, α_2 and δ such that

$$\alpha_1 \|c\|^2 \leqslant \int_t^{t+\delta} |S_\zeta^{\mathrm{T}}(\tau) c|^2 \, \mathrm{d}\mu(\tau) \leqslant \alpha_2 \|c\|^2. \tag{3}$$

holds for any $t \geqslant 0$ and every constant vector $c \in \mathbb{R}^m$.

Based on Refs. [18,23], the following lemma is given in Refs. [15,16], characterizing the PE property for RBF networks.

Lemma 1 (Partial PE condition for RBF NN)**.** *Consider any continuous periodic orbit $Z(t) : [0, \infty) \to \mathbb{R}^q$ with period T_0. $Z(t)$ remains in a bounded compact set Ω_z with $\Omega_z \subset \mathbb{R}^q$. Then, RBF networks $W^T S(Z)$ with centers placed on a regular lattice* (*large enough to cover compact set Ω_z*) *and regress subvector $S_\zeta(Z(t))$ defined in* (2), *is persistently exciting almost always in the sense of* (3).

2.2. Deterministic learning

A simple second-order nonlinear system with unit control gain is considered in [15]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + u \end{aligned} \tag{4}$$

where $x = [x_1, x_2]^{\mathrm{T}} \in \mathbb{R}^2, u \in \mathbb{R}$ are the state variable and control input, respectively. $f(x)$ is a nonlinear function, smooth but unknown.

To make state x in (4) track to a periodic reference orbit $x_d(t)$ generated by the following reference model:

$$\begin{aligned} \dot{x}_{d1} &= x_{d2} \\ \dot{x}_{d2} &= f_d(x_d) \end{aligned} \tag{5}$$

where $x_d = [x_{d1}, x_{d2}]^{\mathrm{T}} \in \mathbb{R}^2$ is the state variable and $f_d(x_d)$ is a smooth nonlinear function, an adaptive neural controller was given by

$$u = -z_1 - c_2 z_2 - \widehat{W}^{\mathrm{T}} S(x) + \dot{\alpha} \tag{6}$$

where $z_1 = x_1 - x_{d1},\ z_2 = x_2 - \alpha,\ \alpha = -c_1 z_1 + x_{d2}.\ c_1, c_2 > 0$ are control gains. $\widehat{W}$ is the estimation of W^*, and is updated by

$$\dot{\widehat{W}} = \dot{\tilde{W}} = \Gamma[S(x) z_2 - \sigma \widehat{W}] \tag{7}$$

where $\widetilde{W} = \widehat{W} - W^*, \Gamma = \Gamma^{\mathrm{T}} > 0$ is a design matrix, and $\sigma > 0$ is of small value.

With the adaptive neural controller and the NN weight updating law, for any periodic orbit $\phi_d \triangleq \{x_d(t) : t \geqslant 0\}$ starting from initial state $x_d(0) \in \Omega_d$, and with $x(0) \in \Omega_0, \widehat{W}(0) = 0$, it holds that [15]: (i) all signals in the closed-loop system remain uniformly ultimately bounded; (ii) the tracking error $\tilde{x}(t) = x(t) - x_d(t)$ converges to an arbitrarily small neighborhood of zero in finite time T_0; (iii) the neural weight estimation $\widehat{W}_\zeta$ converges to a small neighborhood of its optimal value W_ζ^* exponentially in finite time $T_1 \geqslant T_0$, and locally-accurate approximation of $f(x)$ along trajectory ϕ_ζ is obtained as follows:

$$f(x) = \overline{W}_\zeta^{\mathrm{T}} S_\zeta(x) + \varepsilon_{\zeta 2}(x) = \overline{W}^{\mathrm{T}} S(x) + \varepsilon_2(x), \forall x \in \phi_\zeta \tag{8}$$

where

$$\overline{W} = \mathrm{mean}_{t \in [t_a, t_b]} \widehat{W}(t) \tag{9}$$

with $[t_a, t_b]$, $t_b > t_a \geqslant T_1$, and both $|\varepsilon_{\zeta 2}|$ and $|\varepsilon_2|$ are of small values.

2.3. Uniform complete observability [17]

Consider the linear time-varying system $[C(t), A(t)]$ defined by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) \\ y(t) &= C(t)x(t) \end{aligned} \tag{10}$$

where $x(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^m$, while $A(t) \in \mathbb{R}^{n \times n}, C(t) \in \mathbb{R}^{m \times n}$ are piecewise continuous functions. $[C(t), A(t)]$ is called uniformly completely observable (UCO) if there exist strictly positive constants β_1, β_2, δ, such that

$$\beta_2 |x(t_0)|^2 \geqslant \int_{t_0}^{t_0+\delta} |C(\tau)x(\tau)|^2 \mathrm{d}\tau \geqslant \beta_1 |x(t_0)|^2 \tag{11}$$

for all $x(t_0) \in \mathbb{R}^n, t_0 \geqslant 0$, where $x(t)$ is the solution of (10).

The conception of UCO and some lemmas in Ref. [17] will be used to analyze stability and convergence of the error systems in the next section.

3. Main result

3.1. Problem formulation

Consider the following system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + u + d(t) \\ y &= x_1 \end{aligned} \tag{12}$$

where $x = [x_1, x_2]^{\mathrm{T}} \in \mathbb{R}^2$ is the state variable, $u \in \mathbb{R}$ is the system input, $d(t)$ is the uniformly bounded disturbance, $f(x)$ is an unknown smooth nonlinear function. Since $f(x)$ is unknown, an RBF neural network $W^{*\mathrm{T}} S(x)$ is used to approximate it in a compact set $\Omega_x \subset \mathbb{R}^2$ with $\varepsilon(x)$ being the approximation error.

For the convenience of our discussion, some standard assumptions are given as follows.

Assumption 1. There exists an unknown small constant $\varepsilon^* > 0$ such that the function approximation error $\varepsilon(x)$ satisfies $|\varepsilon(x)| \leqslant \varepsilon^*$ for all $x \in \Omega_x$.

Assumption 2. There exists a constant $d^* > 0$ such that $|d(t)| \leqslant d^*$ with d^* not necessarily known.

In the sequel, a second-order reference model is described as follows:

$$\begin{aligned} \dot{x}_{di} &= f_{di}(x_d), i = 1, 2 \\ y_d &= x_{d1} \end{aligned} \tag{13}$$

where $x_d = [x_{d1}, x_{d2}]^{\mathrm{T}} \in \mathbb{R}^2$ is the state variable and $f_{di}(x_d)$, $i = 1, 2$, is a known smooth nonlinear function. The orbit of the reference model starting from the initial state $x_d(0)$ is denoted as $\phi_d(x_d(0))$ (also as ϕ_d for concise presentation). Without loss of generality, assume that the state of the reference model is uniformly bounded in a known compact set, i.e., $x_d(t) \in \Omega_d, \forall t \geqslant 0$, and ϕ_d is a periodic motion.

The objective is to design an adaptive neural controller such that stability and tracking performance are guaranteed, and to analyze the effects of the bounded disturbances to the convergence of RBF neural weights and to the RBF network approximation of uncertain system dynamics.

3.2. Adaptive neural control design

In this subsection, an adaptive neural control scheme will be given such that the output of system (12) can track the output of reference model (13) with the tracking error arbitrarily small.

The adaptive neural controller is given by

$$u_a = -z_1 - \tilde{c}_2 z_2 - \widehat{W}^{\mathrm{T}} S(x) + \dot{\alpha}_a \tag{14}$$

where

$$\begin{aligned} z_1 &= x_1 - x_{d1}, z_2 = x_2 - \alpha_a \\ \alpha_a &= -\tilde{c}_1 z_1 + f_{d1}(x_d) \end{aligned}$$

$\tilde{c}_1, \tilde{c}_2$ are the control gains which will be determined in the latter. $\widehat{W}$ is the estimation of W^* and updated by

$$\dot{\widehat{W}} = \dot{\widetilde{W}} = \Gamma[S(x) z_2 - \sigma \widehat{W}] \tag{15}$$

where $\widetilde{W} = \widehat{W} - W^*$ denotes the estimation error, $\Gamma = \Gamma^{\mathrm{T}} > 0$ is a design matrix, and $\sigma > 0$ is any given constant of small value.

Consider the following Lyapunov function candidate

$$V_a = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \widetilde{W}^{\mathrm{T}} \Gamma^{-1} \widetilde{W} \tag{16}$$

The time derivative of V_a along (12) and (13) satisfies

$$\dot{V}_a = -\tilde{c}_1 z_1^2 - \tilde{c}_2 z_2^2 + \widetilde{W}^{\mathrm{T}} \Gamma^{-1} [\dot{\widehat{W}} - \Gamma S(x) z_2] + d(t) z_2 + \varepsilon(x) z_2$$

Let $\tilde{c}_2 = \sum_{j=0}^{2}\tilde{c}_{2j}$ with $\tilde{c}_{2j} > 0$. In view of the following inequalities

$$-\sigma \widetilde{W}^{\mathrm{T}}\widehat{W} \leqslant -\frac{\sigma\|\widetilde{W}\|^2}{2} + \frac{\sigma\|W^*\|^2}{2}$$
$$-\tilde{c}_{21}z_2^2 + \varepsilon(x)z_2 \leqslant \frac{\varepsilon^2(x)}{4\tilde{c}_{21}} \leqslant \frac{\varepsilon^{*2}}{4\tilde{c}_{21}}$$
$$-\tilde{c}_{22}z_2^2 + d(t)z_2 \leqslant \frac{d^2(t)}{4\tilde{c}_{22}} \leqslant \frac{d^{*2}}{4\tilde{c}_{22}}$$

and update law (15), it yields

$$\dot{V}_a \leqslant -\tilde{c}_1 z_1^2 - \tilde{c}_{20}z_2^2 - \frac{\sigma\|\widetilde{W}\|^2}{2} + \frac{\sigma\|W^*\|^2}{2} + \frac{\varepsilon^{*2}}{4\tilde{c}_{21}} + \frac{d^{*2}}{4\tilde{c}_{22}}. \tag{17}$$

For any arbitrarily small constant $\delta_1 > 0$, by properly selecting $\tilde{c}_{21}, \tilde{c}_{22}, \sigma$, we have

$$\delta_1 \geqslant \frac{\sigma\|W^*\|^2}{2} + \frac{\varepsilon^{*2}}{4\tilde{c}_{21}} + \frac{d^{*2}}{4\tilde{c}_{22}}. \tag{18}$$

Sequentially, from (17) and (18), one has

$$\dot{V}_a \leqslant -\tilde{c}_1 z_1^2 - \tilde{c}_{20}z_2^2 - \frac{\sigma\|\widetilde{W}\|^2}{2} + \delta_1. \tag{19}$$

Then, it is clear that $\dot{V}_a$ is negative definite whenever $|z_1| > \sqrt{\delta_1/\tilde{c}_1}, |z_2| > \sqrt{\delta_1/\tilde{c}_{20}}, \|\widetilde{W}\| > \sqrt{2\delta_1/\sigma}$. Since $\delta_1 > 0$ is any arbitrarily small constant, by choosing $\tilde{c}_1 > 0, \tilde{c}_{20} > 0$ properly, (19) implies that z_1, z_2 and $\widetilde{W}$ are uniformly ultimately bounded (UUB).

For the purpose of achieving that z_1 and z_2 can approach any small neighborhood of zero in finite time, the following Lyapunov function candidate is imposed

$$V_z = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{20}$$

By using the same controller (14) with different control gains, i.e.,

$$u_z = -z_1 - \bar{c}_2 z_2 - \widehat{W}^{\mathrm{T}}S(x) + \dot{\alpha}_z \tag{21}$$

with

$$z_1 = x_1 - x_{d1}, z_2 = x_2 - \alpha_z$$
$$\alpha_z = -\bar{c}_1 z_1 + f_{d1}(x_d)$$

the time derivative of V_z along (12) and (13) satisfies

$$\dot{V}_z = -\bar{c}_1 z_1^2 - \bar{c}_2 z_2^2 - \widetilde{W}^{\mathrm{T}}S(x)z_2 + d(t)z_2 + \varepsilon(x)z_2.$$

Let $\bar{c}_2 = \sum_{j=0}^{3}\bar{c}_{2j}$ with $\bar{c}_{2j} > 0$, in light of the following inequalities

$$-\bar{c}_{21}z_2^2 - \widetilde{W}^{\mathrm{T}}S(x)z_2 \leqslant \frac{\|\widetilde{W}\|^2\|S(x)\|^2}{4\bar{c}_{21}} \leqslant \frac{\tilde{\omega}^{*2}s^{*2}}{4\bar{c}_{21}}$$
$$-\bar{c}_{22}z_2^2 + \varepsilon(x)z_2 \leqslant \frac{\varepsilon^2(x)}{4\bar{c}_{22}} \leqslant \frac{\varepsilon^{*2}}{4\bar{c}_{22}}$$
$$-\bar{c}_{23}z_2^2 + d(t)z_2 \leqslant \frac{d^2(t)}{4\bar{c}_{23}} \leqslant \frac{d^{*2}}{4\bar{c}_{23}}$$

it yields

$$\dot{V}_z \leqslant -\bar{c}_1 z_1^2 - \bar{c}_{20}z_2^2 + \frac{\tilde{\omega}^{*2}s^{*2}}{4\bar{c}_{21}} + \frac{\varepsilon^{*2}}{4\bar{c}_{22}} + \frac{d^{*2}}{4\bar{c}_{23}} \tag{22}$$

where $\tilde{\omega}^*, s^*$ are obtained the same as in Ref. [15].

Since $\tilde{\omega}^*, s^*, \varepsilon^*, d^*$ are positive constants, for any arbitrarily small constant $\delta_2 > 0$, by properly selecting $\bar{c}_{21}, \bar{c}_{22}, \bar{c}_{23}$, we have

$$\delta_2 \geqslant \frac{\tilde{\omega}^{*2}s^{*2}}{4\bar{c}_{21}} + \frac{\varepsilon^{*2}}{4\bar{c}_{22}} + \frac{d^{*2}}{4\bar{c}_{23}}.$$

Hence, (22) can be described as

$$\dot{V}_z \leqslant -\bar{c}_1 z_1^2 - \bar{c}_{20}z_2^2 + \delta_2.$$

By choosing $\bar{c}_1 = \bar{c}_{20} = \gamma/2$ with $\gamma > 0$ any positive constant, it holds that

$$\dot{V}_z \leqslant -\gamma V_z + \delta_2. \tag{23}$$

Let $\rho \triangleq \delta_2/\gamma > 0$, integrating both sides of (23), it yields

$$0 \leqslant V_z(t) \leqslant \rho + (V_z(0) - \rho)\exp(-\gamma t). \tag{24}$$

From (24), one obtains

$$\sum_{i=1}^{2} z_i^2 \leqslant 2\rho + 2V_z(0)\exp(-\gamma t).$$

For any $\mu > \sqrt{2\rho}$, there exists a finite time T_1 such that, for all $t > T_1$, both z_1 and z_2 satisfy

$$|z_i(t)| \leqslant \mu, i = 1, 2. \tag{25}$$

Remark 1. From the design process of adaptive neural control, it is easy to see that two Lyapunov functions and two corresponding controllers are used. The first one is used to guarantee the UUB of all signals in the closed loop systems and the second one is used to make z_i converge to a small neighborhood of zero in finite time T_1. Let $c_1 = \max\{\tilde{c}_1, \bar{c}_1\}$, $c_2 = \max\{\tilde{c}_2, \bar{c}_2\}$, the controller is given by

$$u = -z_1 - c_2 z_2 - \widehat{W}^{\mathrm{T}}S(x) + \dot{\alpha} \tag{26}$$

where

$$z_1 = x_1 - x_{d1}, z_2 = x_2 - \alpha$$
$$\alpha = -c_1 z_1 + f_{d1}(x_d)$$

with adaptive law (15), these results can be obtained simultaneously.

Remark 2. By using controller (26) with adaptive law (15), for any $t > T_1$, one has $\|\widetilde{W}\|^2 \leqslant 2\delta/\sigma$, and thus, it holds that $\|\widehat{W}\|^2 \leqslant 2\delta/\sigma + \|W^*\|^2$.

3.3. Deterministic learning with the PE condition

From (25), it is easy to see that, for any $t \geqslant T_1, z_1 = x_1 - x_{d1}$ is close to zero. Then, x_1 is as periodic as x_{d1} for all $t \geqslant T_1$. At the same time, as $x_2 = z_2 - c_1 z_1 + f_{d1}(x_d), z_1, z_2$ are small and $f_{d1}(x_d)$ is periodic, x_2 is also a periodic-like motion for all $t \geqslant T_1$. Therefore, the partial PE condition of neural network $\widehat{W}^{\mathrm{T}}S(x)$ is satisfied [15]. For the purpose of studying the learning process of $\widehat{W}^{\mathrm{T}}S(x)$, consider the following closed-loop system:

$$\begin{bmatrix} \dot{z} \\ \dot{\widetilde{W}} \end{bmatrix} = \begin{bmatrix} \mathrm{A}_1 & -bS^{\mathrm{T}}(x) \\ \Gamma S(x)b^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} z \\ \widetilde{W} \end{bmatrix} + \begin{bmatrix} b(d(t)+\varepsilon(x)) \\ -\sigma\Gamma\widehat{W} \end{bmatrix} \tag{27}$$

where $z=[z_1,z_2]^{\mathrm{T}},\ A_1=[-c_1,1;-1,-c_2],\ b=[0,1]^{\mathrm{T}}$.

According to the analysis in Ref. [15], by using the localized property of the Gaussian RBF network, decomposing (27), it yields

$$\begin{bmatrix} \dot{z} \\ \dot{\widetilde{W}}_\zeta \end{bmatrix} = \begin{bmatrix} \mathrm{A}_1 & -bS_\zeta^{\mathrm{T}}(x) \\ \Gamma_\zeta S_\zeta(x)b^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} z \\ \widetilde{W}_\zeta \end{bmatrix} + \begin{bmatrix} b(d(t)+\varepsilon'_\zeta(x)) \\ -\sigma\Gamma_\zeta\widehat{W}_\zeta \end{bmatrix} \tag{28}$$

and

$$\dot{\widehat{W}}_{\bar{\zeta}} = \dot{\widetilde{W}}_{\bar{\zeta}} = \Gamma_{\bar{\zeta}}(S_{\bar{\zeta}}(x)b^{\mathrm{T}}z - \sigma\widehat{W}_{\bar{\zeta}}) \tag{29}$$

where $S_\zeta(x)$ is the subvector of $S(x)$ which satisfies the PE condition, $\varepsilon'_\zeta(x)=\varepsilon_\zeta(x)-\widehat{W}_{\bar{\zeta}}^{\mathrm{T}}S_{\bar{\zeta}}(x)$ is the localized NN approximation error along trajectory ϕ_x.

Remark 3. System (27) is referred to as the closed-loop error system. Similar to (55) in Ref. [15], system (28) without the last term is called the nominal part and the last term is named as the disturbed term. It is important to notice that the main difference between system (28) and (55) in Ref. [15] is whether the disturbed term contains $d(t)$. The disturbed term of (55) in Ref. [15] is sufficiently small, and so the convergence of $\widetilde{W}_\zeta$ to a small neighborhood of zero can be established according to Lemma 5.2 in Ref. [19]. In system (27), (28), $d(t)$ is bounded (not small enough), which makes the analysis of parameter convergence much more challenging.

To analyze the stability and convergence property of system (28), the UCO technique [17] and the partial PE condition will be used. In the following, a UCO lemma of $[C, A(t)+K(t)C]$ is presented, with $A(t), C, K(t)$ given by

$$\begin{aligned} A(t) &= \begin{bmatrix} \mathrm{A}_1 & -bS_\zeta^{\mathrm{T}}(x) \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} \bar{C} & 0 & \cdots & 0 \end{bmatrix} \\ K(t) &= \begin{bmatrix} 0 \\ \Gamma_\zeta S_\zeta(x) \end{bmatrix}, \bar{C} = [0 \quad 1]. \end{aligned} \tag{30}$$

Lemma 2. *Consider system $[C, A(t)+K(t)C]$, where $S_\zeta(x)$ satisfies the PE, with the elements of $S_\zeta(x)$ Gaussian RBFs. Then, it holds that*

(i) *$[C, A(t)+K(t)C]$ is UCO, i.e., system (31) is UCO.*
(ii) *System (31) is exponentially stable.*

$$\begin{aligned} \dot{z} &= A_1 z - bS_\zeta^{\mathrm{T}}(x)\widetilde{W}_\zeta \\ \dot{\widetilde{W}}_\zeta &= \Gamma_\zeta S_\zeta(x)b^{\mathrm{T}}z \\ s(t) &= C\chi(t) \end{aligned} \tag{31}$$

where $\chi(t)=col(z(t),\widetilde{W}_\zeta(t))$.

Proof. Since $S_\zeta(x)$ is the PE, for some $\delta>0$ and any $t>0$, there exist $\alpha'_1,\alpha'_2>0$, such that

$$\alpha'_1 \leqslant \int_t^{t+\delta} \|S_\zeta(x(\tau))\|^2 \mathrm{d}\tau \leqslant \alpha'_2.$$

Thus,

$$\int_t^{t+\delta} \|K(\tau)\|^2 \mathrm{d}\tau \leqslant \int_t^{t+\delta} \|\Gamma_\zeta\|^2 \|S_\zeta(x(\tau))\|^2 \mathrm{d}\tau \leqslant \|\Gamma_\zeta\|^2 \alpha'_2 \triangleq k_\delta.$$

In light of Lemma 2.5.2 in Ref. [17], $[C, A(t)+K(t)C]$ is UCO if and only if $[C, A(t)]$ is UCO. Namely, system (31) is UCO if and only if (32) is UCO.

$$\begin{aligned} \dot{z} &= \mathrm{A}_1 z - bS_\zeta^{\mathrm{T}}(x)\widetilde{W}_\zeta \\ \dot{\widetilde{W}}_\zeta &= 0 \\ s'(t) &= \mathrm{C}\chi(t) \end{aligned} \tag{32}$$

It is left to show that system (32) is UCO, i.e., that

$$s'(t) = \overline{\mathrm{C}}e_1^{\mathrm{A}}(t-t_0)z(t_0) + \int_{t_0}^{t} \overline{\mathrm{C}}e_1^{\mathrm{A}}(t-\tau)bS_\zeta^{\mathrm{T}}(x(\tau))\widetilde{W}_\zeta(t_0)\,\mathrm{d}\tau \triangleq s'_1(t)+s'_2(t) \tag{33}$$

satisfies, for some $\beta'_1,\beta'_2,\delta>0$

$$\beta'_1\|\chi(t_0)\|^2 \leqslant \int_{t_0}^{t_0+\delta} s'(\tau)^2 \mathrm{d}\tau \leqslant \beta'_2\|\chi(t_0)\|^2. \tag{34}$$

As S_ζ is the PE and the elements of S_ζ are Gaussian RBFs, let $\mathrm{P}=[1 \quad 0; \quad 0 \quad 1]$, $A_1^{\mathrm{T}}P+PA_1=[-2c_1 \quad 0; 0 \quad -2c_2] \triangleq Q<0, Pb=\overline{C}^{\mathrm{T}}$. In light of Lemma 2.6.7 in Ref. [17], $\int_{t_0}^{t} \overline{C}e^{A_1(t-\tau)}bS_\zeta^{\mathrm{T}}(x(\tau))\,\mathrm{d}\tau$ is the PE for all $t_0 \geqslant 0$. That is, for some $k_1,k_2,v>0$ and any $t_1 \geqslant t_0 \geqslant 0$

$$k_1\|\widetilde{W}_\zeta(t_0)\|^2 \leqslant \int_{t_1}^{t_1+v} s'_2(\tau)^2 \mathrm{d}\tau \leqslant k_2\|\widetilde{W}_\zeta(t_0)\|^2.$$

On the other hand, since A_1 is stable, there exist $\gamma_1,\gamma_2>0$ such that

$$\int_{t_0+mv}^{\infty} s'_1(\tau)^2 \mathrm{d}\tau \leqslant \gamma_1\|z(t_0)\|^2 \mathrm{e}^{-\gamma_2 mv}.$$

Thus, there exists $\gamma_3(mv)>0$ with $\gamma_3(mv)$ increasing along mv such that

$$\int_{t_0}^{t_0+mv} s'_1(\tau)^2 \mathrm{d}\tau \geqslant \gamma_3(m\gamma)\|z(t_0)\|^2$$

for all $t_0, m>0$.

Now, let $n>0$ be another integer and let $\delta=(m+n)v$. In light of inequality $(a+b)^2 \geqslant a^2/2-b^2$, we have

$$\begin{aligned} \int_{t_0}^{t_0+\delta} s'(\tau)^2 \mathrm{d}\tau &\geqslant \frac{1}{2}\int_{t_0}^{t_0+mv} s'_1(\tau)^2 \mathrm{d}\tau - \int_{t_0}^{t_0+mv} s'_2(\tau)^2 \mathrm{d}\tau \\ &\quad + \frac{1}{2}\int_{t_0+mv}^{t_0+\delta} s'_2(\tau)^2 \mathrm{d}\tau - \int_{t_0+mv}^{t_0+\delta} s'_1(\tau)^2 \mathrm{d}\tau \\ &\geqslant \frac{1}{2}\gamma_3(mv)\|z(t_0)\|^2 - mk_2\|\widetilde{W}_\zeta(t_0)\|^2 \\ &\quad + \frac{1}{2}nk_1\|\widetilde{W}_\zeta(t_0)\|^2 - \gamma_1 \mathrm{e}^{-\gamma_2 mv}\|z(t_0)\|^2 \end{aligned}$$

Let m be large enough to get

$$\frac{1}{2}\gamma_3(mv) - \gamma_1 \mathrm{e}^{-\gamma_2 mv} \geqslant \frac{\gamma_3(mv)}{4}$$

and n sufficiently large to obtain

$$\frac{1}{2}nk_1 - mk_2 \geqslant k_1.$$

Further, define

$$\beta_1' = in\left\{k_1, \frac{\gamma_3(mv)}{4}\right\} \tag{35}$$

and similarly, let

$$\beta_2' = \max\{2\gamma_1, 2(m+n)k_2\} \tag{36}$$

Inequality (34) is obtained, thus, system (32) is UCO. According to Lemma 2.5.2 in Ref. [17], let

$$\beta_1 = \beta_1'/(1+\sqrt{k_\delta}\beta_2')^2$$
$$\beta_2 = \beta_2' e^{k_\delta \beta_2'}$$

yields,

$$\beta_1\|\chi(t)\|^2 \leqslant \int_{t_0}^{t_0+\delta} s^2(\tau)\,\mathrm{d}\tau \leqslant \beta_2\|\chi(t)\|^2 \tag{37}$$

That is, system (31) is UCO.

To achieve the exponential stability of system (31), choose the following Lyapunov function candidate

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\widetilde{W}_\zeta^{\mathrm{T}}\Gamma_\zeta^{-1}\widetilde{W}_\zeta \tag{38}$$

The derivative of V along (31) is given by

$$\dot{V} = z_1\dot{z}_1 + z_2\dot{z}_2 + \widetilde{W}_\zeta^{\mathrm{T}}\Gamma_\zeta^{-1}\dot{\widehat{W}}_\zeta \leqslant -c_1z_1^2 - c_2z_2^2 \tag{39}$$

Integrating both sides of (39), it yields

$$\int_t^{t+\delta}\dot{V}\,\mathrm{d}\tau \leqslant -\int_t^{t+\delta}(c_1z_1^2(\tau)+c_2z_2^2(\tau))\,\mathrm{d}\tau$$
$$\leqslant -c_2\int_t^{t+\delta}s^2(\tau)\,\mathrm{d}\tau \leqslant -c_2\beta_1\|\chi(t)\|^2 \tag{40}$$

According to Theorem 1.5.2 in Ref. [17], system (31) is exponentially stable. □

UCO property of system (31) results in its exponential stability. System (31) is the nominal part of the localized error system (28). Now, we focus on the convergence property of system (28) and a lemma is given as follows.

Lemma 3. *Consider system* (28), *where $S_\zeta(x)$ satisfies the PE condition with the elements of $S_\zeta(x)$ Gaussian RBFs. The state variable $\chi(t) = \mathrm{col}(z(t), \widetilde{W}_\zeta(t))$ converges to a neighborhood of zero in finite time $T_2 \geqslant T_1$.*

Proof. Choose the same Lyapunov function as that in Lemma 2, the time derivative of V is given by

$$\begin{aligned}\dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 + \widetilde{W}_\zeta^{\mathrm{T}}\Gamma_\zeta^{-1}\dot{\widehat{W}}_\zeta \\ &\leqslant -c_1z_1^2 - c_2z_2^2 + d(x,t)z_2 + \varepsilon_\varsigma'(x)z_2 + \sigma\widetilde{W}_\varsigma^{\mathrm{T}}\widehat{W}_\varsigma \\ &\leqslant -c_1z_1^2 - (c_{20}+c_{21})z_2^2 + M_1\end{aligned} \tag{41}$$

where $c_2 = c_{20}+c_{21}+c_{22}+c_{23}$ with $c_{2j} > 0,\ j = 0,1,2,3$, and $M_1 = d^{*2}/4c_{22} + \varepsilon^{*2}/4c_{23} + \sigma\|W_\varsigma^*\|^2/2$. It is important to notice that $M_1 > 0$ can be designed small enough by selecting c_{22}, c_{23}, σ properly.

Integrating both sides of (41), it yields

$$\int_t^{t+\delta}\dot{V}\,\mathrm{d}\tau \leqslant -c_{20}\int_t^{t+\delta}z_2^2(\tau)\,\mathrm{d}\tau - \int_t^{t+\delta}(c_1z_1^2(\tau) + c_{21}z_2^2(\tau) - M_1)\,\mathrm{d}\tau. \tag{42}$$

From (42), for any $z \in Z$ with

$$Z = \left\{[z_1, z_2]\,|\,|z_1| > \sqrt{M_1/c_1}, |z_2| > \sqrt{M_1/c_{21}}\right\}$$

it holds that

$$\int_t^{t+\delta}\dot{V}\,\mathrm{d}\tau \leqslant -c_{20}\int_t^{t+\delta}s^2(\tau)\,\mathrm{d}\tau \leqslant -c_{20}\beta_1\|\chi(t)\|^2. \tag{43}$$

Let $\alpha_1 = \min\{\frac{1}{2}, \frac{1}{2}\lambda_{\min}\Gamma_\zeta^{-1}\}, \alpha_2 = \min\{\frac{1}{2}, \frac{1}{2}\lambda_{\max}\Gamma_\zeta^{-1}\}, \alpha_3 = c_{20}\beta_1$, then, similarly to the proof of Theorem 1.5.2 in Ref. [17], for all $|z_1| > \sqrt{M_1/c_1}$ and $|z_2| > \sqrt{M_1/c_{21}}$, one has

$$V(\chi(t+\delta)) \leqslant (1-\alpha_3/\alpha_2)V(\chi(t)), \quad \forall\ t \geqslant 0$$
$$V(\chi(t)) \leqslant V(\chi(t_0+k\delta)), \quad \forall\ t \in [t_0+k\delta, t_0+(k+1)\delta].$$

Choose for t the sequence t_0, $t_0+\delta$, $t_0+2\delta, \ldots$, it yields

$$\begin{aligned}V(\chi(t+k\delta)) &\leqslant (1-\alpha_3/\alpha_2)V(\chi(t_0+(k-1)\delta)) \\ &\leqslant (1-\alpha_3/\alpha_2)^k V(\chi(t_0)) \\ &= m_v \mathrm{e}^{-\lambda_v(k+1)\delta}V(\chi(t_0))\end{aligned} \tag{44}$$

where $m_v = \frac{1}{(1-\alpha_3/\alpha_2)}, \lambda_v = \frac{1}{\delta}\ln\left[\frac{1}{1-\alpha_3/\alpha_2}\right]$.

Consequently, for all $t \geqslant t_0 \geqslant 0$ which satisfies $|z_1(t)| > \sqrt{M_1/c_1}$ and $|z_2(t)| > \sqrt{M_1/c_{21}}$, it holds that

$$V(\chi) \leqslant m_v \mathrm{e}^{-\lambda_v(t-t_0)}V(\chi(t_0)).$$

Similarly,

$$\|\chi(t)\| \leqslant m\mathrm{e}^{-\lambda(t-t_0)}\|\chi(0)\| \tag{45}$$

where $m = \left[\frac{\alpha_2}{\alpha_1(1-\alpha_3/\alpha_2)}\right]^{\frac{1}{2}}, \lambda = \frac{1}{2\delta}\ln\left[\frac{1}{1-\alpha_3/\alpha_2}\right]$.

From (45), $\chi(t)$ exponentially converges with rate $\lambda > 0$. Then, there exists a finite time $T_2 > 0$, for all $t \geqslant T_2, |z_1(t)| \leqslant \sqrt{M_1/c_1}$ and $|z_2(t)| \leqslant \sqrt{M_1/c_{21}}$. And according to inequality (37), we have

$$\|\widetilde{W}_\zeta(t)\|^2 \leqslant \|\chi(t)\|^2 \leqslant \frac{1}{\beta_1}\int_t^{t+\delta}z_2^2(\tau)\,\mathrm{d}\tau \leqslant \frac{M_1\delta}{c_{21}\beta_1}. \tag{46}$$

Consequently, $\chi(t)$ exponentially approaches a neighborhood of zero U in finite time T_2 with U given by

$$U = \left\{\chi(t)\,\|z_1(t)| \leqslant \sqrt{M_1/c_1}, |z_2(t)| \leqslant \sqrt{M_1/c_{21}}, \|\widetilde{W}_\zeta(t)\| \leqslant \sqrt{\frac{M_1\delta}{c_{21}\beta_1}}\right\} \quad \square$$

Remark 4. It is important to notice that under the PE condition of $S_\zeta(x)$, state variable z and localized weight error $\widetilde{W}_\zeta$ exponentially converge to a neighborhood of zero U in finite time $T_2 > T_1$. The size of the neighborhood increases along with $|d(t)|$ and decreases against c_2, which means that the disturbance may deteriorate the convergence of $\widehat{W}$ to its ideal value W^*, and high control gains can be used to depress the influence of the disturbance for the estimation of W^*.
The mean value of $\widehat{W}(t)$ is used in a time segment after the transient process to estimate W^* as follows:

$$\overline{W} = \mathrm{mean}_{t\in[t_1,t_2]}\widehat{W}(t) \tag{47}$$

where $t_2 > t_1 > T_2$. Once $t > t_2, z$ and $\widetilde{W}_\zeta$ approach a neighborhood of zero, by deterministic learning theory with the localized neural network, the dynamics of $f(x)$ along trajectory ϕ_x is learned and can be described as

$$f(x) = \overline{W}^{\mathrm{T}}S(x) + \bar{\varepsilon}(x). \tag{48}$$

The approximation error $|\bar{\varepsilon}(x)|$ depends not only on d^*, but also on the control gain c_2.

The unknown dynamics of $f(x)$ along trajectory (12) is learned by RBF NN $\widehat{W}^{\mathrm{T}}S(x)$ and stored in constant RBF NN $\overline{W}^{\mathrm{T}}S(x)$. Then, for all $t > t_2$, the controller can be given by

$$u = -z_1 - \breve{c}_2 z_2 - \overline{W}^{\mathrm{T}}S(x) + \dot{\alpha} \tag{49}$$

where

$$z_1 = x_1 - x_{d1}, z_2 = x_2 - \alpha$$
$$\alpha = -c_1 z_1 + f_{d1}(x_d).$$

Consider the following Lyapunov function candidate

$$V_c = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{50}$$

The time derivative of V_c along (12) and (13) with controller (48), we have

$$\dot{V}_c \leqslant -c_1 z_1^2 - \breve{c}_{20} z_2^2 + M \tag{51}$$

where $M = d^{*2}/4\breve{c}_{21} + \bar{\varepsilon}^{*2}/4\breve{c}_{22}$ and $\breve{c}_2 = \breve{c}_{20} + \breve{c}_{21} + \breve{c}_{22}$ with $\breve{c}_{2j} > 0, j = 0,1,2$. Then, by properly selecting $c_1, \breve{c}_2$, state z_1 and z_2 stay in a small neighborhood of zero.

Consequently, the following results are obtained.

Theorem 1. *Consider the systems consisting of plant* (12), *reference model* (13), *controller* (26) *with adaptive law* (15). *For any periodic orbit $\phi_d(x_d(0))$ of the reference systems starting from the initial condition $x_d(0) \in \Omega_d, x(0) \in \Omega_x$ and $\widehat{W}(0) = 0$ with Ω_d and Ω_x some compact sets, one has*

(i) *All signals in the closed-loop systems are UUB.*
(ii) *The tracking error $e = y - y_d$ exponentially converges to a small neighborhood of zero in finite time $T_1 > 0$ by appropriately choosing design parameters.*
(iii) *The localized neural-weight estimation $\widehat{W}_\zeta$ converges to a neighborhood U of its optimal value W_ζ^* exponentially in finite time $T_2(> T_1)$ with the size of U increasing along with $|d(t)|$ and decreasing against c_2. Approximation of system dynamics $f(x)$ along orbit ϕ_x is achieved as shown by* (48).
(iv) *By using controller* (49) *with proper control gains, stability of the closed loop systems and good tracking performance are guaranteed.*

Remark 5. Learning control has been studied for a long time. Many interesting approaches including repetitive learning control [24], reinforcement learning control [25], and iterative learning [26] have been proposed. Comparing with these learning control approaches which mainly focus on control problems, one distinguished feature of deterministic learning lies in that it can acquire fundamental knowledge of system dynamics from the stable closed-loop control process. The knowledge can be effectively represented and be reused in closed-loop control to achieve stability and improved control performance.

4. Illustrative example

Consider the following system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + u + d(t) \end{aligned} \tag{52}$$

where $f(x) = -x_1 + 0.7(1 - x_1^2)x_2$ is assumed to be unknown. $d = 0.5\sin(t)$ is time varying disturbance. The desired trajectory $y_d = x_{d1}$ to be tracked is generated by the following Duffing oscillator [27]:

$$\begin{aligned} \dot{x}_{d1} &= x_{d2} \\ \dot{x}_{d2} &= -a_1 x_{d1} - a_2 x_{d1}^3 - a_3 x_{d2} + b\cos(\omega t) \end{aligned} \tag{53}$$

where $x_d = [x_{d1}, x_{d2}]^{\mathrm{T}}$ is the state variable, a_1, a_2, a_3 are parameters. For $a_1 = -1.1, a_2 = 1.0, a_3 = 0.4, \omega = 1.8, b = 1.498$, the trajectory of the Duffing oscillator approaches a period-2 limit cycle with initial state $[x_{d1}(0), x_{d2}(0)]^{\mathrm{T}} = [0.2, 0.3]^{\mathrm{T}}$.

The RBF NN $\widehat{W}^{\mathrm{T}}S(x)$ is used to learn dynamics $f(x)$ along the trajectory of (52), it contains 961 nodes (i.e., $N = 961$), and centers $\xi_i(i = 1,\ldots,N)$ are evenly spaced on $[-3.0, 3.0] \times [-3.0, 3.0]$ with width $\eta_i = 0.2$ ($i = 1,\ldots,N$). Along the control design procedure, the controller is given as follows

$$\begin{aligned} u &= -z_1 - c_2 z_2 - \widehat{W}^{\mathrm{T}}S(x) + \dot{\alpha} \\ \dot{\widehat{W}} &= \dot{\widetilde{W}} = \Gamma[S(x)z_2 - \sigma\widehat{W}] \end{aligned} \tag{54}$$

where $z_1 = x_1 - x_{d1}, z_2 = x_2 - \alpha, \alpha = -c_1 z_1 + x_{d2}$.

Choose the parameters and the initial states in the closed-loop system as $c_1 = 3, c_2 = 15, \Gamma = \mathrm{diag}(5.0), \sigma = 0.002, \widehat{W}(0) = 0, x_i(0) = 0, i = 1,2$. The following simulation results are obtained.

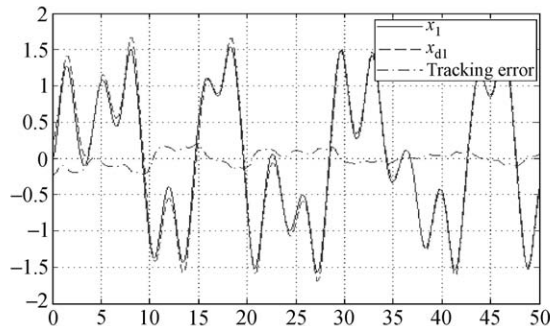

Fig. 1. The tracking performance of x_1 with x_{d1}.

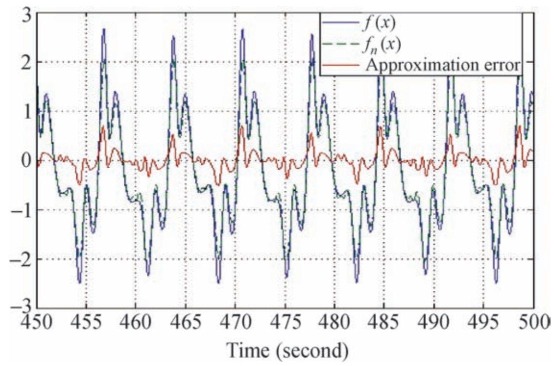

Fig. 4. Learned dynamics of $f(x)$ along the trajectory.

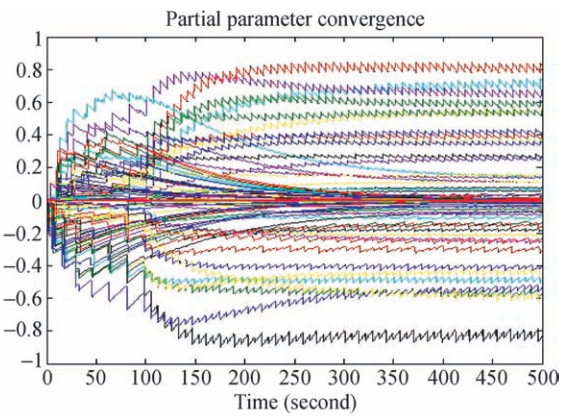

Fig. 2. Partial parameter convergence of $\widehat{W}$.

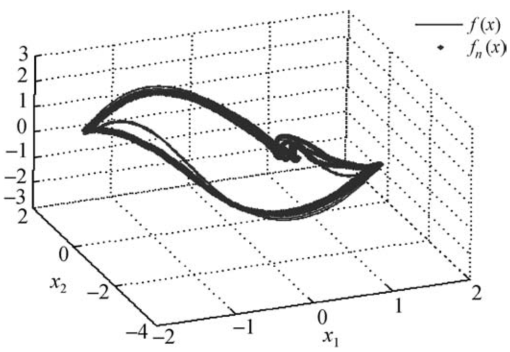

Fig. 5. Approximation along period-2 orbit.

First, the period-2 signal is employed as the reference signal for training the RBF network. Fig. 1 shows the tracking performance of the output of system (52) to the output of reference model (53) in the learning process. The parameter convergence is shown in Fig. 2, it is easy to see that, due to time varying disturbance, the weights are varying around their ideal values in some sense. Fig. 3 shows that the weights along the trajectory are excited sufficiently and those far away from the trajectory are hardly excited. From Figs. 4 and 5, one can see the good learning capability of NN along the trajectory, where $f_n(x) = \overline{W}^{\mathrm{T}}S(x)$. $\overline{W}$ is obtained by (47) with $t_1 = 450, t_2 = 500$. Fig. 6 illustrates the knowledge representation. It shows that the NN approximation by $\overline{W}^{\mathrm{T}}S(x)$ is only accurate in the vicinity of the period-2 orbit, rather than within the entire space of interest. Secondly, Figs. 7 and 8 show the control input given in (49) and the tracking performance, respectively, with $c_1 = 2, c_2 = 3$.

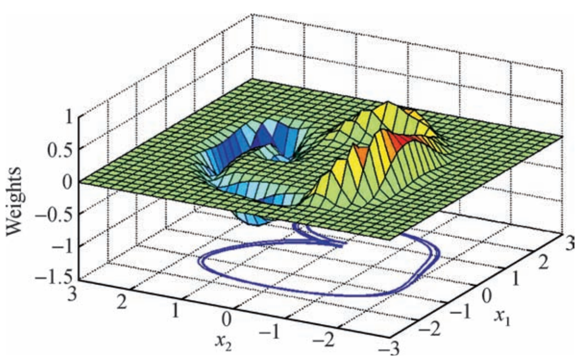

Fig. 3. Weights excited along the trajectory.

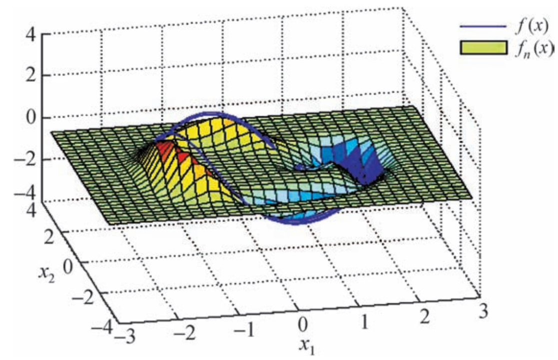

Fig. 6. Approximation in space.

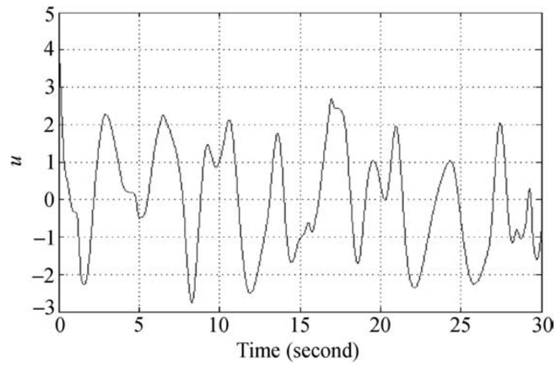

Fig. 7. Control input u of (49).

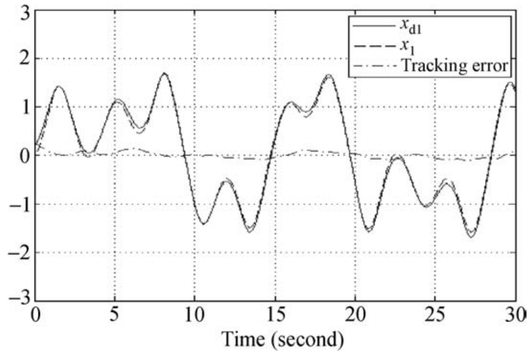

Fig. 8. Tracking performance under controller (49).

5. Conclusion

In this paper, we have shown that for a class of uncertain nonlinear systems with bounded disturbances, deterministic learning can still be implemented. By using an appropriately designed adaptive neural controller and the uniform complete observability (UCO) technique, it has been analyzed that a partial PE condition is satisfied, and partial estimated NN weights converge to a neighborhood of zero, with the size of the neighborhood depending on the amplitude of disturbances as well as on the control gains. Locally-accurate approximation of unknown system dynamics can still be achieved, with the approximation error level influenced by the amplitude of disturbances. The obtained knowledge of system dynamics can be reused to achieve stability and improved performance.

Acknowledgements

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